

Holography and Emergent 4D Gravity

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Abstract

I review recent work toward constructing, via five-dimensional holographic duals, four-dimensional theories in which spin-2 states (gravitons) are emergent. The basic idea is to extend to gravity model-building the applications of holographic duality to phenomenology construction.

1 Introduction

This article is a brief review of the work Ref. [1], in which we applied the ideas of *AdS/CFT* to model-building of theories with modified gravity in four dimensions.

The *AdS/CFT* correspondence [2, 3, 4] establishes an equivalence between a higher-dimensional, gravitational theory on a curved background, and a lower-dimensional field theory, without dynamical gravity.

The basic question I will address here is, *how can one obtain dynamical gravity also on the field theory side*, i.e. in the *lower dimensional* theory? More specifically, is it possible to use the bulk theory as a model for observed gravity ?

The interest in this question arises because one could hope to use holography to construct a UV-complete theory of gravity, as well as consistent modified gravity theories in four-dimensions, e.g. models which are UV-complete and contain massive or metastable gravitons. Models with such properties are difficult to construct directly in four-dimensions, and often suffer from inconsistencies. These difficulties could be bypassed by defining models with the desired properties via holographic duals, which propagate *ordinary* Einstein gravity in higher dimensions.

A simple case of a five/four duality in which the 4D theory contains dynamical gravity is the Randall-Sundrum (RS) model [5, 6] (see Refs. [7, 8, 9, 10] for its holographic interpretation). The 5D theory is defined in a cut-off Anti-de Sitter (*AdS*) space-time. The role of the 4D graviton is played by a normalizable zero-mode, whose profile in the radial direction of *AdS*₅ is peaked in the UV region, as shown in figure 1 (a). This signals the fact that the 4D graviton in RS is a fundamental degree of freedom.

The holographic dual of the RS model, therefore, does have four-dimensional dynamical gravity, but this is achieved trivially by adding the graviton as a fundamental degree of

freedom. In particular, in this model the high energy behavior of 4D gravity is the standard one.

It would be much more interesting to have a situation like the one sketched in figure 1 (b), in which the graviton zero-mode profile is peaked in the IR region: in this case, by standard holography arguments, the four-dimensional graviton would be seen as a composite, rather than a fundamental degree of freedom, and gravity would be an emergent manifestation of some strong, *non-gravitational* IR dynamics. In such a situation, the graviton will cease to exist at high energy, making gravity soft in the UV. This would be a concrete realization of the composite graviton ideas of Ref. [11, 12], and the “fat graviton” ideas of Ref. [13]

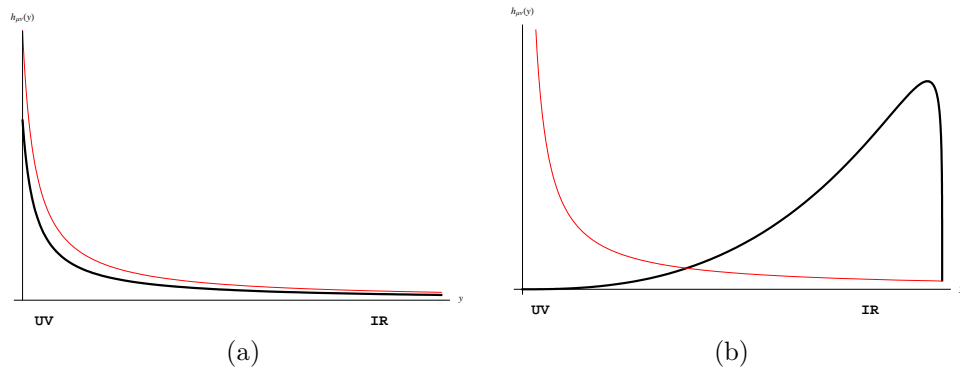


Figure 1: A sketch of the graviton profile (thick black curve) in two possible holographic realizations of four-dimensional gravity: (a) the graviton zero-mode profile in RS2, as a function of the AdS radial coordinate y ; (b) how the same profile would look like if the graviton were a composite state. In both figures the thin red curve is a sketch of the scale factor of the 5D background. The UV and IR correspond to the large and small scale factor regions, respectively.

The problem of obtaining emergent gravity in 4D via holography was addressed in a very general class of five-dimensional models in Ref. [1]. There, we assumed a bottom-up approach, and used the “phenomenological” version of the holographic setup in which, rather than a fully string-theoretical construction, the holographic model consists of Einstein gravity (plus eventually other fields) in a warped five-dimensional background. This approach is not as rigorous as the “stringy” *AdS/CFT* correspondence, nevertheless it is believed to be a valid tool in model building (see Ref. [14] for a recent review).

This work is organized as follows. In Section 2, I review the ideas that underlie the holographic approach to phenomenology. In Section 3, I discuss 4D graviton states in RS-like models. Sections 4 to 8 are devoted to the main results obtained in Ref. [1]: in Sections 4 and 5 I discuss the possibility of obtaining emergent massless 4D spin-2 modes from generic asymptotically *AdS*₅ space-times; Section 6 discusses lower spin states; in Section 7 I briefly discuss how to include matter in the setup; a critical analysis of the results and possible generalizations are discussed in Section 8.

In this article I will omit most of the calculations. The reader is referred to Ref. [1] for all the details, and for a more complete list of references.

2 Holography and Phenomenology

After the original *AdS/CFT* conjecture, it was realized that holography can be effectively used for model building, and people started thinking about extra-dimensional models in terms of holographic duality [7, 8, 9, 10]

The starting point for “holographic phenomenology” is the following. According to gauge/gravity duality, the particle content on the field theory side can be read off as the spectrum of normalizable¹ fluctuation modes around the gravity dual background [3]. We will take the latter to be a 4+1-dimensional space-time, labeled by coordinates (x^μ, y) . Then, a normal mode $\phi_{\{\alpha\}}(x^\mu, y)$ (where $\{\alpha\}$ denotes a set of quantum numbers), such that

$$\square_4 \phi_{\{\alpha\}}(x^\mu, y) = m^2 \phi_{\{\alpha\}}(x^\mu, y), \quad (2.1)$$

corresponds to a state of mass m in the dual gauge theory, with the same quantum numbers $\{\alpha\}$.

The normalizability requirement for a given mode means essentially that one can “dimensionally reduce” the action for that mode by integrating over y , and obtain a finite 4D effective kinetic term. This condition corresponds to normalizability of the wave function in an appropriate quantum mechanical problem.

In the holographic approach to phenomenology, one can therefore “engineer” the 5D space-time in such a way as to obtain a 4D spectrum with desired features. In this language, the five-dimensional description of the model is more tractable than the four-dimensional one, which is that of a (generically unknown) strongly coupled gauge theory. The 5D fluctuation equations compute directly the spectrum of gauge-invariant states, and effectively the 4D model is defined through its 5D dual (see Ref. [14] for a recent review and list of references).

The simplest example is given by the Randall-Sundrum (RS) models [5, 6], in which gravity and eventually other fields live in a slice of *AdS*₅ space,

$$ds^2 = \frac{1}{(ky)^2} (dy^2 + \eta_{\mu\nu} dx^\mu dx^\nu), \quad y_{UV} < y < y_{IR}. \quad (2.2)$$

The coordinate y is dual to the inverse energy scale in 4D, $E \sim y^{-1}$; the small and large y regions correspond to the UV and the IR, respectively of the dual 4D theory; the boundaries y_{UV} and y_{IR} correspond to UV and IR cut-offs, and break conformal invariance; different choices of boundary conditions for the various fields yield different 4D spectra.

One important question, when one considers a particular fluctuation mode, regards its localization properties: modes whose profile is peaked in the small y region are dual to elementary states in the 4D picture; on the other hand, modes that are peaked in the IR correspond to composite states. This distinction is used in model building, for example, to construct models in which various SM particles have different compositeness properties, by localizing them in different regions of the *AdS* slice.

In an asymptotically *AdS*₅ space (i.e. when the UV cut-off is removed, $y_{UV} = 0$, and the theory is UV-complete), normalizable states typically vanish in the UV, and are therefore composite. In general, any mode has an expansion, near the *AdS* boundary $y = 0$, of the form:

$$\phi(x, y) \sim y^{\Delta_-} \phi_-(x) + y^{\Delta_+} \phi_+(x) + \dots, \quad y \rightarrow 0, \quad (2.3)$$

¹In a sense to be made more precise below.

where $\Delta_- < \Delta_+$ depend on the equation governing the particular fluctuation. Typically ϕ_- -modes are not normalizable, and correspond to external sources in the 4D dual FT, while Φ_+ -modes are normalizable (around $y = 0$) and correspond to IR modifications of the theory.

Here, we are interested in 4D states that have the same quantum numbers of physical gravitons in 4D, i.e. transverse and traceless symmetric tensors under the 4D Lorentz group. These can be naturally identified, in the gravity dual, with the tensor fluctuations $h_{\mu\nu}(x, y)$ of the components along space-time of the 5D metric, satisfying:

$$\partial^\mu h_{\mu\nu} = h_\mu^\mu = 0, \quad (2.4)$$

i.e. the 5D “gravity waves.” The aim of our investigation can be summarized in the following question:

Are there asymptotically AdS_5 backgrounds that support normalizable tensor zero-modes ($\square_4 h_{\mu\nu}(x, y) = 0$) ?

The interest in requiring AdS_5 asymptotics in the UV is that, in this way, the 4D dual theory is automatically UV-complete, and free of elementary gravitons, as we will see in more detail in the next Section. Therefore, if a spin-2 zero mode exists, it must correspond to a composite state in a UV-complete non-gravitational theory.

3 Spin-2 states in RS-like models

Let us examine this question in the simplest possible model: pure AdS_5 (better, its Poincaré patch) with no boundaries, i.e. the metric given in (2.2) with $y_{UV} = 0$ and $y_{IR} \rightarrow +\infty$. In this metric, the equation for the transverse traceless metric fluctuations, defined by $\delta g_{\mu\nu} = (ky)^{-2} h_{\mu\nu}$, is [5]:

$$h_{\mu\nu}'' - \frac{3}{y} h_{\mu\nu}' + \square_4 h_{\mu\nu} = 0. \quad (3.1)$$

Zero mode solutions have the general form

$$h_{\mu\nu}(x, y) = h_{\mu\nu}^{(0)}(x) + y^4 h_{\mu\nu}^{(4)}(x), \quad \square_4 h_{\mu\nu}^{(0)}(x) = \square_4 h_{\mu\nu}^{(4)}(x) = 0. \quad (3.2)$$

In the AdS/CFT language, the perturbation $h_{\mu\nu}^{(0)}$ corresponds to turning on a source for the stress tensor in the UV, whereas $h_{\mu\nu}^{(4)}$ corresponds to an IR deformation that gives the stress tensor a VEV.

We can check normalizability of $h_{\mu\nu}^{(0)}$ and $h_{\mu\nu}^{(4)}$ by expanding Einstein’s action to second order around the AdS solution, and isolating the kinetic terms for the tensor fluctuations:

$$S_{kin}[h^{(0)}] = \int_0^{+\infty} \frac{dy}{(ky)^3} \int d^4x \left(\partial_\mu h_{\mu\nu}^{(0)} \right)^2, \quad (3.3)$$

$$S_{kin}[h^{(4)}] = \int_0^{+\infty} \frac{dy y^4}{(ky)^3} \int d^4x \left(\partial_\mu h_{\mu\nu}^{(4)} \right)^2. \quad (3.4)$$

Clearly the y -integrals diverge for both $h_{\mu\nu}^{(0)}$ and $h_{\mu\nu}^{(4)}$: the former is non-normalizable in the UV, the latter in the IR. This model does not contain Spin-2 zero-modes. The fact that $h_{\mu\nu}^{(0)}$ is not UV-normalizable is consistent with the fact that it corresponds to an external non-dynamical source in the UV.

It is easy to modify the above set-up in order to obtain a normalizable tensor zero-mode: it is enough to introduce a UV cut-off, and restrict the fifth dimension to the region $y \geq y_{UV} > 0$. In this way we obtain the RS2 model, in which the mode $h_{\mu\nu}^{(0)}$ is normalizable and mediates four-dimensional gravity. In the dual description, we have made the external source dynamical by adding to it a kinetic term in the fundamental UV Lagrangian.

Thus, In the RS2 model we did obtain a four-dimensional graviton, but in a trivial way: we have added the graviton as an extra fundamental degree of freedom to the theory. A much more radical picture would result, were we able to make the mode $h_{\mu\nu}^{(4)}$ normalizable: since this is peaked in the IR, the resulting state would be interpreted as a composite graviton. We can make the question asked at end of the previous section more precise, and ask:

Are there asymptotically AdS_5 backgrounds that support normalizable zero-modes of the type $h_{\mu\nu}^{(4)}$?

To make $h_{\mu\nu}^{(4)}$ normalizable, the UV cut-off is irrelevant (and we will always remove it from now on), but we have to modify the theory in the IR. The simplest thing one can do is to introduce a cut-off in the IR, and to impose appropriate boundary conditions at y_{IR} such that they are satisfied by $h_{\mu\nu}^{(4)}$. This is a particular case of the more general models constructed by Gherghetta, Peloso and Poppitz [15], that consider a slice of AdS_5 bounded by both IR and UV boundaries, and introduce Pauli-Fierz mass terms both on the branes and in the bulk to control which combination of the graviton zero-modes is in the spectrum.

The approach taken in Ref. [15] is not completely satisfying, as it is subject to the general critique that applies to holographic models with a hard IR cut-off: the fifth dimension terminates abruptly, and not by a dynamical process. In other words, the strong coupling dynamics that leads to the IR scale is completely disconnected from the UV (which in fact has unbroken conformal invariance).

Instead, we would like to consider more general models, in which deviations from AdS in the IR occur dynamically and can be obtained using the 5D field equations. This general analysis was performed in Ref. [1], and a review of the results found there will be the subject of the rest of the present work.

4 Searching for Spin-2 States in General 5D Backgrounds

We aim at investigating the existence of normalizable spin 2 zero-modes in the most general five-dimensional, asymptotically AdS geometry that respects four-dimensional Lorentz invariance. In conformal coordinates ² the metric reads:

$$ds^2 = a^2(y) (dy^2 + \eta_{\mu\nu} dx^\mu dx^\nu), \quad y > 0 \quad (4.1)$$

²I will use 5D coordinates $x^A = \{y, x^\mu\}$; four-dimensional indices will always be contracted with the flat metric $\eta_{\mu\nu}$; a prime will denote derivative w.r.t. y .

and has an AdS_5 -like boundary with curvature scale k at $y = 0$,

$$a(y) \sim \frac{1}{ky} \quad y \rightarrow 0. \quad (4.2)$$

It is useful, although not strictly necessary if we are only interested in tensor modes, to work with a concrete five-dimensional setup. If we assume the additional requirement that the space-time respects the Null Energy Condition (NEC)³, we can realize any metric of the form (4.1) as a solution of an Einstein-Dilaton system with an appropriate potential. Therefore, we take the 5D dynamics to be described by the action:

$$S = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-g} \left(R - g^{AB} \partial_A \Phi \partial_B \Phi - V(\Phi) \right), \quad (4.3)$$

The background solution is specified by the scale factor $a(y)$ and by the scalar field profile $\Phi(y)$. Choosing an appropriate potential $V(\Phi)$ one can obtain any metric of the form (4.1) that satisfies the NEC (see Refs. [18, 19]). AdS asymptotics require that, as $y \rightarrow 0$:

$$\Phi(y) \rightarrow \Phi_0, \quad V(\Phi_0) = 2\Lambda \equiv -12k^2. \quad (4.4)$$

Consider now tensor fluctuations around the background metric, defined by:

$$ds_{pert}^2 = a^2(y) \left[dy^2 + (\eta_{\mu\nu} + h_{\mu\nu}(x, y)) dx^\mu dx^\nu \right], \quad h_\mu^\mu = \partial^\mu h_{\mu\nu} = 0. \quad (4.5)$$

The action (4.3), expanded to second order in $h_{\mu\nu}$ around the solution, reads:

$$\begin{aligned} S^{(2)}[h_{\mu\nu}] &= -\frac{1}{8\kappa_5^2} \int dy d^4x \sqrt{-g} g^{AB} \partial_A h_{\mu\nu} \partial_B h^{\mu\nu} \\ &= -\frac{1}{8\kappa_5^2} \int dy d^4x a^3(y) \left[(h'_{\mu\nu})^2 + (\partial_\rho h_{\mu\nu})^2 \right]. \end{aligned} \quad (4.6)$$

The corresponding linearized field equation is:

$$h''_{\mu\nu} + 3\frac{a'}{a}h'_{\mu\nu} + \square_4 h_{\mu\nu} = 0. \quad (4.7)$$

Notice that it is only the scale factor that enters this equation, not the scalar field profile. Therefore, the properties of the tensor modes depend only on the background metric, and not on the details of the underlying model of which it is a solution. We can change the matter content by adding more fields, or replace the scalar field by e.g. a perfect fluid, but all conclusions about these modes will still hold. These details *do* matter for other modes, like scalar and vector modes. I will briefly discuss these lower spin fluctuations in Section 6.

The problem of finding solutions with a given 4D mass can be reduced to a one-dimensional quantum mechanical problem in the y -coordinate. To see this, assume a factorized ansatz:

$$h_{\mu\nu}(x, y) = h(y) h_{\mu\nu}^{(4d)}(x), \quad \square_4 h_{\mu\nu}^{(4d)}(x) = m^2 h_{\mu\nu}^{(4d)}(x), \quad (4.8)$$

³This is the weakest of the energy conditions, and dropping it typically results in pathological behavior such as violation of causality and presence of ghosts.

and define the “wave-function” $\Psi(y)$ as:

$$\Psi(y) = e^{-B(y)} h(y), \quad B(y) \equiv -\frac{3}{2} \log a(y). \quad (4.9)$$

In these variables, the tensor mode equation (4.7) becomes the Schrödinger-like equation:

$$-\Psi'' + [(B')^2 - B''] \Psi = m^2 \Psi. \quad (4.10)$$

As in the previous section, the normalizability condition can be read-off from the quadratic action (4.6):

$$S \sim \int dy e^{-2B} h^2(y) \int d^4x \left(\partial_\rho h_{\mu\nu}^{(4d)} \right)^2 = \left(\int dy |\Psi|^2 \right) \int d^4x \left(\partial_\rho h_{\mu\nu}^{(4d)} \right)^2. \quad (4.11)$$

Requiring finiteness of the effective four-dimensional kinetic term for $h_{\mu\nu}^{(4d)}(x)$, imposes the standard normalizability condition on the wave-function $\Psi(y)$, i.e. it must be square-integrable.

The problem of finding spin-2 zero modes is thus reduced to finding the zero-energy eigenstates, $\mathcal{H}\Psi = 0$, of the Hamiltonian

$$\mathcal{H} = -\partial_y^2 + \mathcal{V}(y), \quad \mathcal{V}(y) \equiv (B')^2 - B''. \quad (4.12)$$

on the Hilbert space of square-integrable functions $\Psi(y)$. The Hamiltonian is completely specified by the scale factor $a = e^{-2B/3}$, and we want to determine what are the backgrounds such that zero-energy eigenstates exist.

While in general it is not possible to determine explicitly the spectrum of \mathcal{H} , for any choice of $B(y)$ the two independent zero-energy solutions of the differential equation (4.10) can be written explicitly, and are given by:

$$\Psi^{UV}(y) = e^{-B(y)}, \quad \Psi^{IR}(y) = e^{-B(y)} \int_0^y dy' e^{2B(y')}. \quad (4.13)$$

The integration constant in the second solution has been chosen such that Ψ^{IR} vanishes at $y = 0$: recall that *AdS* asymptotics require $B(y) \sim -3/2 \log y$ as $y \rightarrow 0$, therefore close to the *AdS* boundary:

$$\Psi^{UV}(y) \sim y^{-3/2}, \quad \Psi^{IR}(y) \sim y^{5/2}, \quad y \rightarrow 0. \quad (4.14)$$

Thus, Ψ^{IR} is normalizable around $y = 0$, whereas Ψ^{UV} is not⁴.

Up to now, we have one candidate spin-2 zero mode, normalizable in the UV, whose profile is given by Ψ^{IR} in eq. (4.13). Whether or not this is indeed an eigenstate of \mathcal{H} depends on the behavior of $B(y)$ in the IR, i.e. for large y . From the holographic perspective, having zero-modes in the spectrum depends on the infrared dynamics. Notice that, by definition, Ψ^{IR} vanishes in the UV, so the corresponding state, if it exists, is composite rather than fundamental, since it ceases to exist at high energy.

As a first step in the analysis, one can separate two classes of backgrounds:

⁴Both become normalizable if, as in RS, we introduce a cut-off and restrict $y > 1/\Lambda$.

1. the y coordinate ranges from 0 to $+\infty$;
2. space-time ends at a singularity or a boundary at a finite y_0 .

It is fairly easy to see that, in the first case, Ψ^{IR} cannot possibly be normalizable as $y \rightarrow \infty$: in fact it even fails to go to zero in that limit, no matter what is the asymptotic behavior of $B(y)$. Thus, from now on we discard this possibility.

In the second class of models, we certainly can have zero modes if we truncate the space with a hard boundary (*hard wall*) at finite y . This is rather artificial, though, and there are more general possibilities: for example the space-time can terminate dynamically by developing a singularity at finite y . An IR singularity, unlike a hard boundary, always arises as the consequence of the running of some coupling (in our case, the non-trivial background scalar field $\Phi(y)$), and it seems to be inevitable in 5D holographic models with a non-trivial IR dynamics (for example, some recently proposed realistic-looking 5D holographic duals of confining gauge theories also have IR singularities [16, 17]).

What kind of singular behavior of $B(y)$ results in a normalizable $\Psi^{IR}(y)$? Suppose, for example, that the singularity is due to the vanishing of the scale factor at $y = y_0$. Then, as $y \rightarrow y_0$, $B(y) \rightarrow +\infty$, and

$$\Psi^{IR}(y) \sim (y_0 - y)e^{B(y)}. \quad (4.15)$$

Requiring Ψ^{IR} to be normalizable around y_0 , we obtain the constraint:

$$\Psi^{IR} \prec \frac{1}{(y_0 - y)^{1/2}} \quad \Rightarrow \quad e^{B(y)} \prec \frac{1}{(y_0 - y)^{3/2}}, \quad (4.16)$$

i.e. $B(y)$ should diverge at most as fast as $-3/2 \log(y_0 - y)$.

The other possibility is that $B(y)$ remains finite at y_0 , and the singularity is due to a divergence of one of its derivatives. In this case Ψ^{IR} has a finite limit and it is trivially normalizable.

Therefore, using the relation $a = e^{-2B/3}$, we can make the following general statement:

In a five-dimensional, asymptotically AdS background, there can be normalizable spin-2 zero-modes if the space-time ends at some finite value y_0 of the conformal coordinate, where the scale factor $a(y)$ either stays finite, or vanishes more slowly than $(y_0 - y)$.

5 The Need for IR Boundary Conditions

The space-times supporting spin-2 zero modes are either singular, or terminated by a hard wall in the IR. The presence of a singularity typically means that the description being used breaks down, and one has to get a better understanding of the infrared dynamics. One can hope that the singularity will be resolved in a full string theory realization. However, it is possible that the results obtained using the singular theory do not depend too much on the details of the dynamics that regularizes the space-time. This is the case, for example, in models when the singularity is “screened” from physical fluctuations: the wave functions do not probe the high curvature region, and any modification in this region will not drastically modify the spectrum of low-lying states [17]. As I will discuss below, in the scenario discussed here we are not so lucky, and it is not possible to completely decouple the singularity.

In gauge/gravity duality, one expects the properties of the field theory to be completely defined by the gravity action, plus a set of boundary conditions in the UV. In the case of a hard wall, we need to supply extra information to specify both the wall position and the IR boundary conditions for the various fields (equivalently, one has the freedom to add boundary terms to the action in the IR). The UV data are not enough to completely determine the spectrum. Since the normalizable wave-functions Ψ^{IR} generically do not vanish in the IR, changing the details of the wall will affect the spectrum.

When the fifth dimension ends in a singularity, on the other hand, the position of the singularity is completely determined by the UV asymptotics. In this case, one can say that space-time ends dynamically. In certain cases, the requirement of normalizability is sufficient to determine the spectrum completely. If this is so, the details of resolution of the singularity do not significantly affect the spectrum⁵.

In other cases however, one still has to provide additional information in the IR, in the form of “boundary conditions” at the singularity, and this is precisely what happens in the singular backgrounds that support normalizable spin-2 zero-modes. Let us go back to the eigenvalue equation, eq. (4.10), and suppose the singularity at $y = y_0$ is of the form:

$$B(y) \sim -\alpha \log(y_0 - y), \quad a(y) \sim (y_0 - y)^{2\alpha/3}, \quad \alpha \geq 0 \quad (5.1)$$

As we have seen in Section 3, the zero-modes Ψ^{IR} are normalizable if and only if $0 < \alpha < 3/2$ ⁶. For any m^2 , close to the singularity eq (4.10) reads approximately:

$$-\Psi'' - \frac{\alpha^2 - \alpha}{(y_0 - y)^2} \Psi \sim 0, \quad (5.2)$$

with the two independent solutions:

$$\Psi \sim c_1(y_0 - y)^\alpha + c_2(y_0 - y)^{1-\alpha}, \quad (5.3)$$

for arbitrary coefficients c_1 and c_2 . For $0 < \alpha < 3/2$, i.e. when spin-2 zero-modes are allowed, *both solutions are normalizable*. This means that normalizability at the singularity does not impose any restriction on the eigenfunctions, and a solution to the *full* equation exists for *any* value of m^2 (in particular $m^2 = 0$). This does not mean that the model has a continuous spectrum. Rather, the spectral problem is not fully specified by the information supplied so far.

From a more formal point of view, the spectral problem is not fully specified, because the Hamiltonian (4.12) is not essentially self-adjoint. Rather, it is a symmetric operator possessing infinite non-equivalent self-adjoint extensions, each with a different spectrum. Picking one of these extensions is equivalent to specifying the asymptotic behavior of the solutions at y_0 , i.e. by fixing the ratio of the coefficients c_1 and c_2 appearing in (5.3). There exists one specific choice for which the zero-mode is in the spectrum, but generically this will not be the case.

⁵In the same sense that, for example, the QCD details of the proton structure are irrelevant for the spectrum of the hydrogen atom

⁶The case $\alpha < 0$ violates the NEC, and cannot be realized in the model (4.3), or in general in any model without ghost-like degrees of freedom.

The situation with the singularity, therefore, is not so different from the hard wall scenario. In both cases there are normalizable, IR-localized spin-2 zero modes only for very special choices of the IR boundary conditions.

Notice that, in the singular case, when $1/2 < \alpha < 3/2$, or more generally when

$$-\log(y_0 - y) \prec B(y) \prec 3/2 \log(y_0 - y),$$

the metric perturbation $h(y) = e^{B(y)}\Psi(y)$ diverges at the singularity, and the linearized analysis cannot be trusted. On the other hand, when $\alpha < 1/2$, the metric perturbation reaches the finite value c_1 at the singularity, so the linear approximation remains reliable.

6 Lower Spin Modes

So far I have discussed the tensor spectrum, to answer the question whether one can realize holographically a scenario where massless gravitons are emergent. However, for the viability of the model, it is important to check whether there also exist massless lower spin states. In particular, we want to exclude models with scalar⁷ zero-modes (such as the radion in RS, or the extra scalar mode in DGP) that couple with gravitational strength, since these are typically bad for phenomenology. Massless transverse vector modes, on the other hand, can be tolerated, since they do not couple to a conserved stress tensor. Here I will only sketch the main results, and refer the reader to Ref. [1] for details.

6.1 Scalars

As we have seen, the spectrum of tensor fluctuations is rather model-independent, and depends only on the background metric. On the other hand the spectrum of scalar fluctuations generically depends on the full background. Moreover, of all the scalar fluctuation one can write, only a certain number corresponds to physical modes, the others can be gauged away using (linearized) diffeomorphisms (whereas tensor fluctuations are gauge-invariant).

In a model with a single scalar coupled to gravity, such as (4.3) one can show that, in the massive sector, there is a single physical scalar fluctuation, which solves a Schrödinger equation similar to eq. (4.10), but with $B \rightarrow B - \log[a\Phi'/a']$. In the massless sector things are different due to an enhanced gauge invariance, and as shown in Ref. [1] there are two independent fluctuations ζ_1, ζ_2 , which solve the equations:

$$\zeta_1' = 0; \quad \left(\frac{a^4}{a'} \zeta_2 \right)' = -2a^3 \zeta_1. \quad (6.1)$$

The most general scalar zero mode, then, has the form:

$$\zeta_1(x, y) = F(x); \quad \zeta_2 = \frac{a'}{a^4}(y)G(x) - 2 \left(\frac{a'}{a^4} \int a^3 \right) (y)F(x) \quad (6.2)$$

where $F(x)$ and $G(x)$ are two arbitrary function satisfying $\square_4 F, G = 0$.

Careful analysis of the effective kinetic term for $F(x)$ and $G(x)$ reveals that F is not UV-normalizable, and G (the would-be radion in the case of RS) is not IR-normalizable around

⁷Here, the terms “scalar” and “vector” refer to the transformation properties under the 4D Lorentz group.

singularities that allow spin 2 zero modes. On the other hand, in hard wall models it must be killed by boundary conditions.

6.2 Vectors

In this sector there are only zero-modes: there do not exist transverse vector fluctuations at the massive level. The extra scalar and vector that appear in the massless sector become, in the massive sector, the three additional degrees of freedom of the massive tensor fluctuations (five d.o.f., as opposed to two for the massless tensor fluctuations). Therefore in each sector we have a total of six physical d.o.f., arranged in different representations of the four-dimensional Lorentz group.

The transverse vector equation is simply:

$$(a^3(y)A_\mu(x, y))' = 0 \quad \Rightarrow \quad A_\mu(x, y) = a^{-3}(y)v_\mu(x). \quad (6.3)$$

The effective action for this mode is the 4D Maxwell action for $v_\mu(x)$, with the prefactor $\int dy a^{-3}(y)$. This mode is UV-normalizable in asymptotically AdS backgrounds, and it is IR normalizable around singularities where that $a(y)$ vanishes more slowly than $(y_0 - y)^{1/3}$. This overlaps, but does not coincide, with the behavior that allows spin-2 zero-modes.

6.3 Summary of Results

The results about the existence or non-existence of zero modes of various spins in IR-singular, UV- AdS space-times is summarized in Table 1.

	$y \in (0, \infty)$	$y \in (0, y_0)$				
$B(y_0)$	—	$-\alpha \log(y_0 - y)$			$finite + (y_0 - y)^\beta$	
		$0 < \alpha < 1/2$	$1/2 < \alpha < 1$	$1 < \alpha < 3/2$	$0 < \beta < 1$	$1 < \beta < 2$
Spin-2	—	○	○	○	○	○
Spin-1	—	○	—	—	○	○
Spin-0	—	—	—	—	—	○

Table 1: 4D massless spectrum as a function of the IR behavior of $B(y)$. A line — or a circle ○ indicate absence or presence, respectively, of the corresponding normalizable zero-mode. For the parameter ranges not shown in the table there is no normalizable spin-2 zero-mode. The last three lines indicate the behavior of the Shrödinger potential, the tensor mode wave-function and the scalar curvature near the end of the y -coordinate range.

As remarked in Section 5, the case $1/2 < \alpha < 3/2$ is problematic, since close to the singularity the metric perturbations diverge.

7 Coupling Emergent Gravitons to 4D Matter

In the setup described above, four-dimensional matter can be coupled to gravity by adding a probe 3-brane at a fixed position y_b , where the Standard Model is localized. If we couple

ordinary matter to the induced metric on the brane,

$$S_{matter} = \int d^4x \sqrt{-g_{ind}} \mathcal{L}_{SM}, \quad (7.1)$$

the 5D action for linearized gravity plus localized matter will be:

$$S = -\frac{1}{8\kappa_5^2} \int dy \frac{a^3(y)}{a^2(y_b)} (\partial_\rho h_{\mu\nu}(x, y))^2 + \int_{y=y_b} h_{\mu\nu}(y_b) T^{\mu\nu}, \quad (7.2)$$

where we have rescaled to physical coordinates, $x^\mu \rightarrow a^{-2}(y_b)x^\mu$, so that the background metric on the brane is $\eta_{\mu\nu}$.

The effective 4D gravitational coupling will depend on the graviton profile evaluated at $y = y_b$. Inserting the definition,

$$h_{\mu\nu}(x, y) = a^{-3/2}(y) \Psi(y) h_{\mu\nu}^{(4d)}(x), \quad (7.3)$$

we get from (7.2) the effective action for the four-dimensional graviton $h_{\mu\nu}^{(4d)}(x)$ coupled to matter:

$$S = -\left(\frac{1}{8k_5^2 a^2(y_b)} \int dy |\Psi|^2 \right) \int d^4x \left(\partial_\rho h_{\mu\nu}^{(4d)} \right)^2 + a^{-3/2}(y_b) \Psi(y_b) \int d^4x h_{\mu\nu}^{(4d)} T^{\mu\nu}. \quad (7.4)$$

The effective four-dimensional Newton's constant can be read-off from the above equation (using unit-norm wave-functions):

$$\sqrt{8\pi G_N} = k_5 \frac{\Psi(y_b)}{\sqrt{a(y_b)}} \quad (7.5)$$

The gravitational coupling vanishes as y_b^3 as $y_b \rightarrow 0$, and placing the SM brane close enough to the AdS_5 boundary, $y = 0$, the four-dimensional Planck scale can be made parametrically larger than the typical Kaluza-Klein masses, which is generically of order $1/y_0^2$.

8 Generalizations

We have seen that, using holographic constructions, one does not generically obtain IR-localized, normalizable massless spin-2 modes in four-dimensions. These modes exist only for a small class of IR singularities, and only if special boundary conditions are imposed at the end of space. These singularities are not of the “good” kind, since they are not screened from propagating modes, and the spectrum heavily depends on the details of the singularity.

Nevertheless, one can investigate if there exist constructions that result in a regularized version of these singularities, and whether one can (more or less naturally) obtain the right boundary conditions that yield massless gravitons in the regularized models. One step in this direction could be the string theory constructions described in Ref. [20], which start from continuous distributions of D3-branes in type IIB, and upon reduction to 5D yield precisely a singularity of the right kind (e.g. one can get a scale factor that behaves as in (5.1), with $\alpha = 1/2$).

The difficulties we encountered are not surprising, in view of the Weinberg-Witten theorem [21]. This states that, in four dimensions, one cannot have massless spin-2 modes that couple universally to a Lorentz-covariant stress tensor⁸. What we are looking for after all, via a holographic detour, is composite massless spin-2 states in a strongly coupled field theory defined by its gravity dual, so we are running against the Weinberg-Witten theorem.

However, it is not obvious from the start that the Weinberg-Witten theorem should apply to these holographic constructions, since the IR-localized graviton zero modes, when it exists, will not couple universally to the fundamental degrees of freedom. The source for the field theory stress tensor is encoded in the non-normalizable mode in the UV, and this is the mode that couples universally to the stress tensor at all energies. An example is the zero-mode in the Randall-Sundrum models, which has a constant profile, and the universality of its coupling in 4D descends from the same property in 5D. On the other hand, the UV-normalizable mode cannot interact as a 4D graviton at all energy scales. In fact, it becomes soft and decouples from the fundamental theory at high energy, (in particular the high energy behavior of these gravitons violates the equivalence principle).

Although the Weinberg-Witten theorem does not straightforwardly apply to our case, one can guess that some of the difficulties we have encountered can be traced to that argument. In order to alleviate these difficulties, therefore, one can try to lift some of the hypotheses of that theorem, and at the same time make less stringent requirements on the “emergent” spin-2 states we want to construct. There are (at least) two possible roads one can follow:

1. Giving up masslessness, and looking for 5D space-times with normalizable spin-2 states that have a tiny 4D mass.
2. Giving up normalizability, and allowing the graviton to be a very long lived resonance.

Interestingly, both ideas (massive and/or metastable gravitons) have received a lot of attention from the phenomenological point of view, but at the same time present several unresolved theoretical challenges. The holographic approach can probably lead, for the first time, to manifestly consistent realizations of these kinds of models.

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⁸Ordinary gravity evades this theorem because the gravitational stress tensor does not transform covariantly under a Lorentz transformation (it does only up to a (linearized) diffeomorphism, which mixes its helicity 2 components with those of lower helicity).

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